MTH 201: Multivariable Calculus and Differential Equations

Homework V

(Due 22/10)

1. Write an iterated integral for $\iint_A dA$ over the region R bounded by:

   (a) $y = \tan x$, $x = 0$, and $y = 1$.

   (b) $y = x^2$ and $y = x + 2$.

   (c) $y = e^{-x}$, $y = 1$, and $x = \ln 3$.

2. Sketch the region of integration and evaluate the following integrals.

   (a) $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dx\, dy$.

   (b) $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$.

   (c) $\int_{-\pi/3}^{\pi/3} \int_0^{\sec t} 3\cos t\, du\, dt$.

   (d) $\int_0^{3/2} \int_1^{4-2u} \frac{4-2u}{v^2}\, dv\, du$.

3. Sketch the region of integration, reverse the order of integration, and evaluate.

   (a) $\int_0^{3/2} \int_0^{9-4x^2} 16x\, dy\, dx$.

   (b) $\int_0^{\pi/6} \int_{\sin x}^{1/2} xy^2\, dy\, dx$.

   (c) $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3y\, dx\, dy$.

   (d) $\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy$.

   (e) $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dy\, dx$.

   (f) $\int_0^{1/16} \int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\, dx\, dy$.

4. Find the volume of the region bounded above by the surface $z = f(x, y)$ and below by the region R.

   (a) $f(x, y) = y - \sqrt{x}$, R: Bounded by $x \geq 0$, $y \geq 0$, and $x + y = 1$.

   (b) $f(x, y) = x^2$, R: Bounded by $y = 2 - x^2$ and $y = x$ in the xy-plane.

   (c) $f(x, y) = \sqrt{4 - x^2}$, R: The smaller sector cut from the disk $x^2 + y^2 \leq 4$ by the rays $\theta = \pi/6$ and $\theta = \pi/2$.

   (d) $f(x, y) = \frac{1}{(x^2-x)(y-1)^{2/3}}$, R: $[2, \infty) \times [0, 2]$.

5. Compute the volume of the following solids.

   (a) The solid in the first octant bounded by the coordinate planes, the plane $x = 3$, and the parabolic cylinder $z = 4 - y^2$.

   (b) The wedge cut from the first octant by the cylinder $z = 12 - 3y^2$ and the plane $x + y = 2$.

   (c) The solid that is bounded front and back by the planes $x = 2$ and $x = 1$, on the sides by the cylinders $y = \pm 1/x$, and above and below by the planes $z = x + 1$ and $z = 0$.

6. Evaluate the following improper integrals.

   (a) $\displaystyle\int_1^{\infty}\int_{e^{-x}}^{1} \frac{1}{x^3 y}\, dy\, dx.$

   (b) $\displaystyle\int_0^{\infty}\int_0^{\infty} x e^{-(x+2y)}\, dx\, dy.$